Quantum Anomaly and Effective Field Description of a Quantum Chaotic Billiard

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(Dated: February 2, 2008)

We investigate the effective field theory of a quantum chaotic billiard from a new perspective of quantum anomalies, which result from the absence of continuous spectral symmetry in quantized systems. It is shown that commutators of composite operators on the energy shell acquire anomalous part. The presence of the anomaly allows one to introduce effective dual fields as phase variables without any additional coarse-graining nor ensemble averaging in a ballistic system. The spectral Husimi function plays a role as the corresponding amplitude.

PACS numbers: 73.23.Ad,05.45.Mt,02.20.-a

The study of quantum chaos, namely, quantum properties of classically nonintegrable systems, has been attracting much attention over two decades. By the presence of irregular boundary and/or impurities inside, a quantum billiard system falls into this category, and it has been a rich source of research not only as a manageable model bridging between chaos and quantum mechanics but also as a model of electronic devices such as quantum dots. With the standard quantization method heavily relying on the presence of invariant tori, the quantization prescription of classically chaotic systems is not yet fully disclosed. Yet the success of quantum mechanics is so striking that it seems sensible to seek what is a quantum signature of classically chaotic systems by assuming the validity of the Schrödinger equation.

The symmetry of quantum theories may be substantially different from the classical one since it is not always possible to retain all the classical symmetries at quantum level. When some classical symmetry is broken, one may call a quantum system anomalous and detect it by the presence of quantum anomalies [1]. Often, quantum anomalies will matter in the context of gauge field theories, yet the notion goes far beyond it and serves as a fundamental feature of quantum theories.

In this Letter, we investigate the effective field description of a quantum chaotic billiard from a novel perspective — quantum anomalies in spectra. The relevance of the anomaly to a quantum billiard is most easily understood by noting different spectral structures between classical and quantum theories. Whereas classical dynamics has continuous symmetry along the energy by changing the momentum continuously without altering the orbit in space, such continuous symmetry is absent quantum mechanically since discrete energy levels are formed. In this sense, the anomaly here is taken as a revelation of the quantization condition. By examining the algebraic structure of the "current algebra", we will show the presence of anomalous part (Schwinger term) in current commutators. Accordingly, it enables us naturally to construct effective fields as phase variables *without* any additional coarse-graining nor ensemble averaging, whereas the spectral Husimi function shows up and acts as the amplitude (see Eqs. (14,17) below).

Effective field theories by the supermatrix nonlinear-sigma (NL-σ) model have been quite successful in describing disordered metals with diffusive dynamics [2]. In addition to explaining weak localization phenomena, the zero-mode approximation has provided a *direct proof* of the Bohigas-Giannoni-Schmit conjecture stating that the level correlations of quantum chaotic systems in general obey the Wigner-Dyson statistics from random matrices [3]. By seeing the success of effective field description of diffusive chaotic systems, the same zero-dimensional model has been anticipated by the universality in a ballistic system without any intrinsic stochasticity nor disorder, and a great effort has been put forth to extend the framework toward it. Over last decade, several "derivations" of ballistic NL-σ models, which is claimed to be applicable to the length scale shorter than the mean free path, have been proposed, based on a quasiclassical approximation [4], on the ensemble averaging either over energy spectra [5, 6] or over the external parameter [7], or by a functional bosonization approach [8]. Meanwhile our understanding has progressed considerably in particular on the connection between the statistical quantum properties and the classical chaotic dynamics.

The validity of these ballistic NL-σ models, however, is not so transparent unlike the diffusive counterpart. Soon after the derivations, it has been recognized that some (unphysical) zero mode exists along a vertical direction of the energy shell and nothing suppresses those fluctuations [6, 7]. As a result, how to attain the necessary "mode-locking" has been disputed. Though similar difficulty is absent in some approaches [4, 8], the mode-locking mechanism in those works is ascribed to assuming the existence of the Fermi surface. This is rather odd because the Fermi surface is a many-body effect and has nothing to do with the notion of quantum chaos. In the present work, we will find an explanation by the anomaly carried by each level. The mechanism is of one-body nature and applicable either to noninteracting bosons or noninteracting fermions.

In general, quantum anomalies can be detected in current commutators by anomalous part (Schwinger term)

proportional to the derivative of the Dirac delta function. There are several ways to reveal such contribution [1]: point-splitting methods, normal ordering prescription, cohomological consideration, examining the functional Jacobian etc. Among them, we choose the normal ordering prescription with the canonical quantization scheme (see also [9]). In contrast to previous effective field theories with functional integration, we find that the present approach clarifies the subtlety hidden in regularization in a clearer and more pedagogical way.

The *intrinsic* need of regularization stems from the presence of discretized energy levels ε_α. In energy integration, the effect is accommodated by an insertion of the Dirac delta function $\delta(\varepsilon-\varepsilon_\alpha)$, but its singular nature requires some regularization to make the theory finite. A standard way is to define the Dirac delta function with positive infinitesimal η by

$$\delta(\varepsilon-\varepsilon_\alpha)=\frac{i}{2\pi}\left(\frac{1}{\varepsilon-\varepsilon_\alpha+i\eta}-\frac{1}{\varepsilon-\varepsilon_\alpha-i\eta}\right). \qquad (1)$$

A crucial observation in the present context is to view it as an embodying the point-splitting method along the energy axis, hence a field theory defined on the energy coordinate is called for. The point-splitting regularization is known to be equivalent to introducing the normal-ordered operator, so that we proceed it by defining the appropriate vacuum $|0\rangle$ and creation/annihilation parts of operators.

Having in mind the Hamiltonian $\mathcal{H}$ describing quantum dynamics in an irregular confinement potential, we begin with the Schrödinger equation in the first quantized form, $\mathcal{H}\phi_\alpha(\boldsymbol{r})=\varepsilon_\alpha\phi_\alpha(\boldsymbol{r})$ with eigen energies ε_α and eigen functions $\phi_\alpha(\boldsymbol{r})$. We consider a *generic* quantum chaotic billiard, by which we mean that neither spectral degeneracy nor dynamical symmetry intertwining the spectra exists. In this situation, it makes sense to attach independent creation/annihilation operators $\psi_\alpha^\dagger$ and ψ_α to each level α. Though operators ψ_α may be either bosonic or fermionic, we first assume them as fermionic, which highlights a similarity of the present construction to the conventional bosonization in one-dimensional electrons. By these preparations, we define the field operator $\psi(\boldsymbol{r})=\sum_\alpha\phi_\alpha(\boldsymbol{r})\psi_\alpha$ obeying the commutation relation $\{\psi(\boldsymbol{r}_1),\psi^\dagger(\boldsymbol{r}_2)\}=\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)$. Since we work on a non-interacting system, we can immediately write down the commutation relation not only at equal time but also at different time. When we further introduce the field operator on a certain energy shell ε by

$$\psi(\boldsymbol{r}t)=\int_{-\infty}^{\infty}\psi(\boldsymbol{r}\varepsilon)\,e^{-\frac{i}{\hbar}\varepsilon t}d\varepsilon, \qquad (2)$$

they are found to satisfy the commutation relation

$$\begin{aligned}&\left\{\psi(\boldsymbol{r}_1\varepsilon_1),\psi^\dagger(\boldsymbol{r}_2\varepsilon_2)\right\}.\\ &\quad=\delta(\varepsilon_1-\varepsilon_2)\,\langle\boldsymbol{r}_1|\,\delta(\varepsilon_2-H)\,|\boldsymbol{r}_2\rangle\,, \end{aligned}\qquad (3)$$

where $\delta(\varepsilon-H)=\sum_\alpha|\alpha\rangle\delta(\varepsilon-\varepsilon_\alpha)\langle\alpha|$ is the spectral operator. The above shows an intriguing feature as a field theory that field operators on the energy shell are nonlocal in space because of the nonlocality of the spectral operator. Simultaneously, apart from nonlocality, the system may be viewed as the one-dimensional system along the "ε-axis" with some internal degrees of freedom attached. We will pursue this line of description below.

The field $\psi(\boldsymbol{r}\varepsilon)$ is a subtle object because it exists when ε coincides with one of eigen energy levels ε_α and divergence occurs due to the Dirac delta function. The latter divergence is regularized by Eq. (1) but the prescription is incomplete for composite fields. The nonlocal commutation relation requires us to consider a nonlocal current operator (or density operator in usual terminology), and we need to define it by normal-ordering by decomposing operators into $(\pm)$ parts, $\psi=\psi_++\psi_-$ [9]. Explicitly, projected field operators $\psi_\pm$ are defined by

$$\psi_\pm(\boldsymbol{r}\varepsilon)=\frac{\pm i}{2\pi}\int_{-\infty}^{\infty}\frac{\psi(\boldsymbol{r}\varepsilon')}{\varepsilon-\varepsilon'\pm i\eta}\,d\varepsilon', \qquad (4)$$

and $\psi_\pm^\dagger$ by its hermitian conjugate. Subsequently we introduce the vacuum state (the Dirac sea) $|0\rangle$ by requiring $\psi_+|0\rangle=\psi_-^\dagger|0\rangle=0$. It means that ψ_+ and $\psi_-^\dagger$ ($\psi_+^\dagger$ and ψ_-) are annihilation (creation) operators and the normal-ordered products $:\,:$ are defined accordingly. The commutation relation Eq. (3) is modified for the projected fields to be

$$\begin{aligned}&\left\{\psi_\pm(\boldsymbol{r}_1,\varepsilon_1),\psi_\pm^\dagger(\boldsymbol{r}_2,\varepsilon_2)\right\}\\ &\quad=\frac{(\pm i/2\pi)}{\varepsilon_1-\varepsilon_2\pm i\eta}\,\langle\boldsymbol{r}_1|\delta(\varepsilon_2-H)|\boldsymbol{r}_2\rangle,\end{aligned}\qquad (5)$$

and all the other commutation relations vanish. By using this prescription, the normal-ordered current operator is defined by

$$j(\boldsymbol{r},\boldsymbol{r}';\varepsilon)=\,:\psi^\dagger(\boldsymbol{r}'\varepsilon)\psi(\boldsymbol{r}\varepsilon):\,. \qquad (6)$$

To see how anomalous contribution emerges in the current commutator, it suffices to examine the vacuum average, which can be evaluated by the help of Eq. (5) and $2\pi\delta'(z)=-(z+i\eta)^{-2}+(z-i\eta)^{-2}$ as

$$\begin{aligned}&\langle 0|\,[j(\boldsymbol{r}_1,\boldsymbol{r}_1';\varepsilon_1),j(\boldsymbol{r}_2,\boldsymbol{r}_2';\varepsilon_2)]\,|0\rangle\\ &\quad=\frac{i}{2\pi}\langle\boldsymbol{r}_2|\delta(\varepsilon_1-H)|\boldsymbol{r}_1'\rangle\langle\boldsymbol{r}_1|\delta(\varepsilon_2-H)|\boldsymbol{r}_2'\rangle\,\delta'(\varepsilon_1-\varepsilon_2).\end{aligned}\qquad (7)$$

The right-hand side signifies anomalous contribution, *i.e.*, Schwinger term. It is present only when the spatial correlation of the spectral operator exists. Having identified the anomalous part, we can immediately restore the current algebra as

$$[j(\boldsymbol{r}_1,\boldsymbol{r}_1';\varepsilon_1),j(\boldsymbol{r}_2,\boldsymbol{r}_2';\varepsilon_2)]$$

$$= \delta(\varepsilon_1 - \varepsilon_2)\left[\langle \boldsymbol{r}_1|\delta(\varepsilon_2 - H)|\boldsymbol{r}_2'\rangle j(\boldsymbol{r}_1', \boldsymbol{r}_2; \varepsilon_2) - (1 \leftrightarrow 2)\right] + \frac{i}{2\pi}\langle \boldsymbol{r}_2|\delta(\varepsilon_1 - H)|\boldsymbol{r}_1'\rangle\langle \boldsymbol{r}_1|\delta(\varepsilon_2 - H)|\boldsymbol{r}_2'\rangle\delta'(\varepsilon_1 - \varepsilon_2). \tag{8}$$

The above determines the current algebra on the energy shell completely, but working on an bilocal operator of the form $\mathcal{O}(\boldsymbol{r}, \boldsymbol{r}')$ is not so convenient. A way to circumvent the difficulty is to recast it into an object defined on the classical phase space $\boldsymbol{x} = (\boldsymbol{q}, \boldsymbol{p})$ by taking it as $\mathcal{O}(\boldsymbol{r}, \boldsymbol{r}') = \langle \boldsymbol{r}|\hat{\mathcal{O}}|\boldsymbol{r}'\rangle$ (still an operator). In previous approaches [4, 5, 6, 7, 8], the Wigner-Weyl representation has been widely utilized with a semiclassical approximation. Nevertheless, we find that the use of the Husimi representation (the wave-packet representation) not only has some advantages but also mandatory to identify the exact symmetry of the algebra.

The Husimi representation of the operator $\hat{\mathcal{O}}$ is defined by $\mathcal{O}(\boldsymbol{x}) = \langle \boldsymbol{x}|\hat{\mathcal{O}}\,|\boldsymbol{x}\rangle$ where the coherent state $|\boldsymbol{x}\rangle$ centered at $\boldsymbol{x} = (\boldsymbol{q}, \boldsymbol{p})$ is defined by

$$\langle \boldsymbol{r}|\boldsymbol{x}\rangle = (\pi\hbar)^{-\frac{d}{4}}\, e^{-\frac{1}{2\hbar}(\boldsymbol{r}-\boldsymbol{q})^2 + \frac{i}{\hbar}\boldsymbol{p}\cdot(\boldsymbol{r}-\frac{1}{2}\boldsymbol{q})}. \tag{9}$$

Note that the coherent basis $|\boldsymbol{x}\rangle$ is overcomplete, so that one can determine the operator uniquely by its diagonal element while it is never the case in ordinary complete bases. Following this convention, we identify the Husimi representation of j as $\langle \boldsymbol{x}|j(\varepsilon)|\boldsymbol{x}\rangle$. The definition can be equally rewritten in terms of the field operator $\psi(\boldsymbol{x}) = \int d\boldsymbol{r}\langle \boldsymbol{x}|\boldsymbol{r}\rangle\psi(\boldsymbol{r})$ annihilating the wave-packet centered at $\boldsymbol{x} = (\boldsymbol{q}, \boldsymbol{p})$ as

$$j(\boldsymbol{x}; \varepsilon) = :\psi^\dagger(\boldsymbol{x}; \varepsilon)\psi(\boldsymbol{x}; \varepsilon):\ . \tag{10}$$

It is checked that the operator $\psi(\boldsymbol{x}; \varepsilon)$ obeys the commutation relation $\{\psi(\boldsymbol{x}; \varepsilon_1), \psi^\dagger(\boldsymbol{x}; \varepsilon_2)\} = \delta(\varepsilon_1 - \varepsilon_2)H(\boldsymbol{x})$ where $H(\boldsymbol{x}; \varepsilon) = \langle \boldsymbol{x}|\delta(\varepsilon - \mathcal{H})|\boldsymbol{x}\rangle$ is the spectral Husimi function. Consequently, the projected fields obeys

$$\left\{\psi_\pm(\boldsymbol{x}; \varepsilon_1), \psi_\pm^\dagger(\boldsymbol{x}; \varepsilon_2)\right\} = \frac{(\pm i/2\pi)H(\boldsymbol{x}),}{\varepsilon_1 - \varepsilon_2 \pm i\eta} \tag{11}$$

By using the above, we can finally write the current algebra Eq. (8) as

$$[j(\boldsymbol{x}; \varepsilon_1), j(\boldsymbol{x}; \varepsilon_2)] = \frac{i}{2\pi}H(\boldsymbol{x}; \varepsilon_1)H(\boldsymbol{x}; \varepsilon_2)\delta'(\varepsilon_1 - \varepsilon_2). \tag{12}$$

This reveals clearly that $j(\boldsymbol{x}, \varepsilon)/H(\boldsymbol{x}, \varepsilon)$ satisfies the Abelian Kac-Moody algebra *exactly.* Now we can complete bosonization (dual field formulation) at each $\boldsymbol{x}$ by introducing chiral boson fields $\varphi(\boldsymbol{x}; \varepsilon)$

$$[\varphi(\boldsymbol{x}; \varepsilon_1), \varphi(\boldsymbol{x}; \varepsilon_2)] = -i\pi\,\mathrm{sgn}(\varepsilon_1 - \varepsilon_2), \tag{13}$$

by rewriting the current as

$$j(\boldsymbol{x}; \varepsilon) = H(\boldsymbol{x}; \varepsilon)\,\partial_\varepsilon\varphi(\boldsymbol{x}; \varepsilon). \tag{14}$$

Note that the dual field φ is meaningful only when the "amplitude" $H(\boldsymbol{x}; \varepsilon)$ does not vanish. They exist only near energy levels and the mode-locking is fulfilled in this sense.

In passing, it is worth pointing out the difference between the Husimi and the Wigner-Weyl representations. Within the latter, it appears possible to derive an algebra similar to Eq. (12) *approximately* by a semiclassical expansion where the spectral Wigner function shows up instead of the spectral Husimi function. However, a wild oscillation of the Wigner function makes such a semiclassical expansion difficult to justify in general. In contrast, the Husimi representation helps us identify the symmetry *exactly.* Moreover since the Husimi function can be viewed as a Gaussian smoothing of the Wigner function, it has a well-defined semiclassical limit as a coarse-grained classical dynamics [10, 11]. It is noted that from a classical point of view, the energy level itself is taken as a caustic because of a vanishing chord [11], so that the present approach effectively extracts information of the time scale longer than the Ehrenfest time.

We are so far concerned only with the symmetry of a single energy shell ε as appears in $\det(\varepsilon - \mathcal{H})$. Since the n-point spectral correlation can be generated from the n-fold ratio of the determinant correlator $\prod_{i=1}^n \det(\varepsilon_{fi} - \mathcal{H})/\det(\varepsilon_{bi} - \mathcal{H})$, we need to take account of additional degrees of freedom: the graded (boson-fermion) symmetry and the internal symmetry among n energy shells. As a result, the relevant symmetry is enlarged to a general linear Lie superalgebra $\mathfrak{g} = \mathfrak{gl}(n|n)$ in the simplest case (the unitary class), for which the preceding treatment can be extended with minimal modification. Explicitly, we introduce the superbracket $[\![\cdot, \cdot]\!]$ and write the commutation/anti-commutation relations for bose/fermion fields as $[\![\psi_\alpha, \psi_\beta^\dagger]\!] = \delta_{\alpha\beta}$. By introducing the wave-packet (super-spinor) field operator $\psi(\boldsymbol{x}; \varepsilon)$, the normal-ordered current operators on the Husimi representation are defined by

$$j_a(\boldsymbol{x}, ; \varepsilon) = :\psi^\dagger(\boldsymbol{x}; \varepsilon)X_a\psi(\boldsymbol{x}; \varepsilon): \tag{15}$$

where $X_a \in i\mathfrak{g}$ is an Hermitian element of a given Lie superalgebra $\mathfrak{g}$ obeying $[\![X_a, X_b]\!] = if_{ab}^c X_c$. One can evaluate the current commutator on the Husimi representation as before to find

$$[\![j_a(\boldsymbol{x}; \varepsilon_1), j_b(\boldsymbol{x}; \varepsilon_2)]\!] = if_{ab}^c\,\delta(\varepsilon_1 - \varepsilon_2)H(\boldsymbol{x}; \varepsilon_2)j(\boldsymbol{x}; \varepsilon_2) + \frac{i\,\kappa_{ab}}{2\pi}H(\boldsymbol{x}; \varepsilon_1)H(\boldsymbol{x}; \varepsilon_2)\delta'(\varepsilon_1 - \varepsilon_2), \tag{16}$$

where $\kappa_{ab} = \mathrm{STr}[X_a X_b]$. This is the main result of the paper. It shows clearly that $j_a(\boldsymbol{x}; \varepsilon)/H(\boldsymbol{x}; \varepsilon)$ satisfies the Kac-Moody algebra of a corresponding Lie superalgebra. Hence the effective field theory is described by the (chiral) Wess-Zumino-Novikov-Witten model defined on a corresponding Lie supergroup with the current operator

$$j(\boldsymbol{x}; \varepsilon) = H(\boldsymbol{x}; \varepsilon)\; g^{-1}\partial_\varepsilon g(\boldsymbol{x}; \varepsilon). \tag{17}$$

By recognizing the convoluted function $(\varepsilon-\varepsilon'\pm i\eta)^{-1}$ in Eq. (4) is a Fourier transform of the step function, the projection onto $(\pm)$ may be regarded as the decomposition into the retarded (R) and advanced (A) components. We can make the correspondence explicit by writing

$$\psi_+(\boldsymbol{x};\varepsilon)=\begin{pmatrix} b_R \\ f_R \end{pmatrix};\quad \psi_-(\boldsymbol{x};\varepsilon)=\begin{pmatrix} -b_A^\dagger \\ f_A^\dagger \end{pmatrix}, \qquad (18)$$

where $b_{R,A}$ ($f_{R,A}$) are taken as bosonic (fermionic) fields to generate the retarded/advanced Green functions. The minus sign of $b_A^\dagger$ is mandatory to retain the commutation relations. The condition of the vacuum state becomes $b_R|0\rangle = f_R|0\rangle = b_A|0\rangle = f_A|0\rangle = 0$ so that the definition of the normal ordering coincides with the standard definition in the bf-fields. From here, one can construct the color-flavor transformation by using the coherent states of bf-fields at each $\boldsymbol{x}$, as is given in Appendix of [12]. Hence the supermatrix NL-σ model. The only modification that is crucial for the mode-locking problem is the presence of the spectral Husimi function $H(\boldsymbol{x};\varepsilon)$ instead of the average DOS as a result of the commutation relation Eq. (11). In this way, the supermatrix NL-σ model with the exact DOS, $h^{-d}\int H(\boldsymbol{x};\varepsilon)d\boldsymbol{x}$, can be derived in a ballistic system. The zero-dimensional approximation leads to the universal Wigner-Dyson correlation, with nonuniversal deviation from the *coarse-grained* semiclassical dynamics of the Husimi function. Further additional coarse-graining or semiclassical approximation gives a smoothing of DOS, which is believed to correspond to the situation argued in [4, 5, 6, 7, 8].

In the present work, we are concerned with the simplest case that the phase space $\boldsymbol{x}$ has no additional symmetry (the unitary class). It is not the case in the time-reversal symmetric system, where both the symmetry of equivalent vacua and the corresponding algebra need to be enlarged by the time-reversal operation, *i.e.*, mixing between an original field and its Hermitian conjugate counterpart. The relevant symmetry of the n-point level correlation in this case is identified as an orthosymplectic algebra $\mathfrak{osp}(2n|2n)$, which coincides with the symmetry of the Bogoliubov transformation of the enlarged space. Any additional discrete/continuous symmetry will be accommodated similarly.

It is stressed that fields appearing in the effective field description result essentially from the phase arbitrariness of each eigen wavefunction, *i.e.*, $U(1)$ symmetry at each level (or the Berry phase). By putting quantum chaos aside, it is worth thinking of the implication of the present construction in interacting electrons, where we no longer have $U(1)$ symmetry to each level but only one global $U(1)$ phase at the Fermi energy. This suggests that the present construction may still be meaningful at the Fermi energy and should be closely related to the Luther-Haldane bosonization for interacting electrons [13] (by including the degeneracy properly). However, an explicit construction toward it is open at present.

In conclusion, we have examined the effective field theory of a quantum chaotic billiard from the perspective of quantum anomalies, and the theory is shown to be endowed with a symmetry of the Kac-Moody algebra exactly. This allows one to formulate the supermatrix NL-σ model without introducing any additional coarse-graining nor stochasticity. It is also found that the use of the Husimi representation is indispensable to identifying the correct symmetry, and the spectral Husimi function acts as the amplitude of the effective fields.

I thank A. Tanaka for helpful comments on bosonization methods.

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